Circuits and Systems

Analysis vs Synthesis

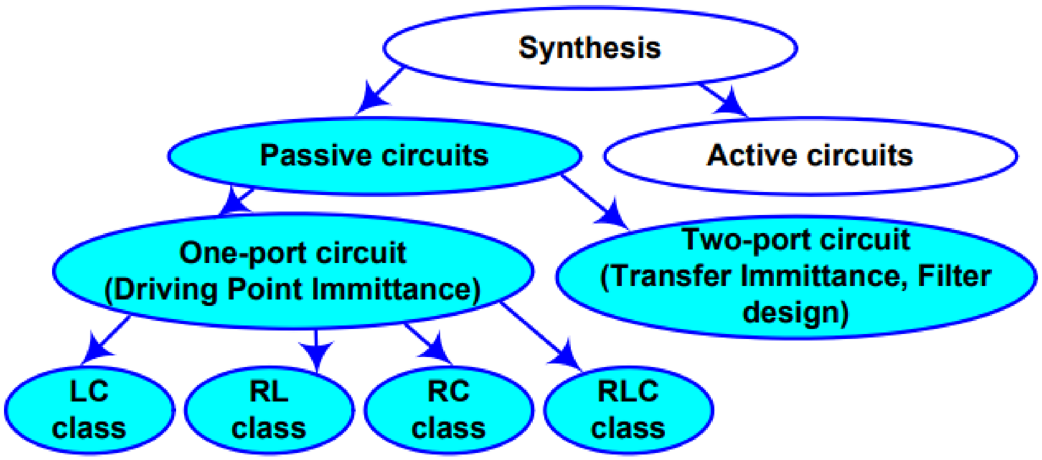
Analysis problem:
Given the structure and the input of the system we determinate its output

The answer of analysis (systematic approach):
There is one answer

Synthesis problem:
Given the input and the output of the system we wish to find its structure

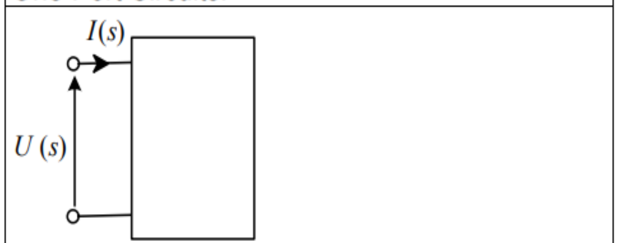
The answer of the synthesis (design approach):
Sometimes there is no answer and often the answer is not unique

The Available Elements for the Synthesis



Synthesis of Passive Circuits

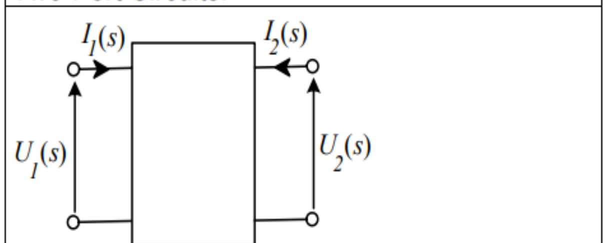
One-Port Circuits:



$\frac{U(s)}{I(s)}$ - driving point impedance

$\frac{I(s)}{U(s)}$ - driving point admitance

Two-Port Circuits:



$\frac{U_1(s)}{I_1(s)}$ - driving point impedance

$\frac{I_1(s)}{U_1(s)}$ - driving point admitance

$\frac{U_2(s)}{I_1(s)}$ - transfer impedance

$\frac{I_2(s)}{U_1(s)}$ - transfer admitance

Additionally:

$\frac{U_2(s)}{U_1(s)}$ - voltage ratio

$\frac{I_2(s)}{I_1(s)}$ - current ratio

The only types of elements allowed are R, L and C (lumped or distributed). The constraints of stability and causality lead to mathematical conditions on the form of Z(s) and Y(s).

Immittance is the word used generically for impedances and admittances. In general, an immittance can be a driving point immittance or a transfer immittance.

Immittance and rational positive real function

A network with finite number of R,L,C elements has a driving point immittance, one-port impedance Z(s) or admittance Y(s), of the **rational** form:

$$F(s)=\frac{N(s)}{D(s)}=\frac{a_n s^n + a_{n-1}s^{n-1}+\ldots+a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1}+\ldots+b_1 s + b_0}$$

Given function F(s) can be **immittance** of design RLC passive circuit (in Z(s) or Y(s) form) only if fulfil requirements for being **rational positive real function**.

Condition for real character of *F(s)* (necessary and sufficient):

C.R.1. coefficient of numerator $(a_n, a_{n-1}, \ldots, a_1, a_0)$ and denominator $(b_m, b_{m-1}, \ldots, b_1, b_0)$ polynomials must be real

Conditions for positive character of *F(s)* (sufficient):

C.P.1. coefficient of numerator $(a_n, a_{n-1}, \ldots, a_1, a_0)$ and denominator $(b_m, b_{m-1}, \ldots, b_1, b_0)$ polynomials must have the same sign

C.P.2. power of "*s*" in numerator and denominator may differ at most by 1 ($(n-m)\subset\{-1,0,1\}$)

C.P.3. Poles and zeros must lie on the left-hand half plane or on the imaginary axis

C.P.4. If the poles and zeros lie on the imaginary axis their must be single (real s=0 or in conjugate pairs $s=\pm j\omega$) and F(s) must have positive, real residues for poles ($\underset{s=s_{pole}}{res}\,F(s)=\lim_{s\to s_{pole}}\left(s-s_{pole}\right)\cdot F(s)$), and positive, real derivative for zeros $F'\left(s_{zero}\right)$

C.P.5. replacing $s=j\omega$ obtained form must have positive real part $\mathrm{Re}\left\{F\left(j\omega\right)\right\}\geq 0$ for $0\leq\omega\leq\infty$

Examples

Verification process of F(s) in point of possible realization as one-port RLC immittance (test of realizability)

Verified function (F(s))	Check list of conditions		
$F(s)=\dfrac{s}{s^2+2s+1}$ power of numerator n=1 zeros: s=0 power of denominator m=2 poles: $s_{1,2}$=-1	C.R.1	all coefficient real	O.K.
	C.P.1	all coefficient with the same sign	O.K.
	C.P.2.	power of "*s*" in numerator and denominator differs by 1 n-m=-1	O.K.
	C.P.3	all zeros and poles lie on the left-hand half plane or on the imaginary axis	O.K.
	C.P.4	only one zero lie on the imaginary axis and its real the derivative in s=0 is real and positive $F'(s=0)=\left.\dfrac{(s^2+2s+1)-2s\cdot s}{(s^2+2s+1)^2}\right\vert_{s=0}=1$	O.K.

$F(s)=\frac{s}{s^2+2s+1}$	C.P.5	$F(j\omega)=\frac{j\omega}{(j\omega)^2+2j\omega+1}=\frac{j\omega}{1-\omega^2+2j\omega}=$ $=\frac{j\omega\cdot\left[\left(1-\omega^2\right)-j2\omega\right]}{\left(1-\omega^2\right)^2+4\omega^2}=\frac{2\omega^2+j\omega\left(1-\omega^2\right)}{\left(1-\omega^2\right)^2+4\omega^2};$ Thus, $\mathrm{Re}\{F(j\omega)\}=\frac{2\omega^2}{\left(1-\omega^2\right)^2+4\omega^2}$ and its real and positive for al $0\le\omega\le\infty$	O.K.
Final decision: Circuit possible for design and realization in RLC passive class			

Verified function (F(s))	Check list of conditions		
$F(s)=\frac{s^2-1}{s^2+3s+2}$ power of numerator n=2 zeros: $s_1=1$, $s_2=-1$ power of denominator m=2 poles: $s_1=-1$, $s_2=-2$	C.R.1	all coefficient real	O.K.
	C.P.1	there is one coefficient which sign differs from the others	NOT O.K.
	C.P.2.		
	C.P.3		
	C.P.4		
	C.P.5		
		Final decision: Circuit can not be design and realize in RLC passive class	

Verified function (F(s))	Check list of conditions		
$F(s)=s+j$ power of numerator n=1 zeros: $s_1=-j$ power of denominator m=0	C.R.1	there is one complex coefficient	NOT O.K.
	C.P.1	all coefficient with the same sign	O.K.
	C.P.2.	power of "*s*" in numerator and denominator differs by 1 n-m=1	O.K.
	C.P.3	all zeros and poles lie on imaginary axis	O.K.
	C.P.4	there is zero s=-j which lies on imaginary axis but hasn't got its conjugate pair s=j	NOT O.K.
	C.P.5		
		Final decision: Circuit can not be design and realize in RLC passive class	

Foster Synthesis

First Foster Form

$$Z(s) = \frac{H(s^2+\omega_1^2)(s^2+\omega_3^2)\ldots(s^2+\omega_m^2)}{(s^2+\omega_2^2)(s^2+\omega_4^2)\ldots(s^2+\omega_r^2)}$$

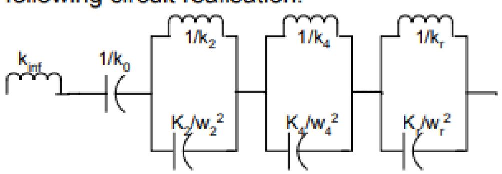
where $0 \le \omega_1^2 \le \omega_2^2 \le \omega_3^2 \le \ldots$

and $m = r \pm 1$

By partial fraction expansion we can write:

$$Z(s) = k_\infty s + \frac{k_0}{s} + \sum_{p=2,4,\ldots}^{r} \frac{k_p s}{s^2+\omega_p^2}, \qquad k_\infty, k_0 \ge 0,\ k_p > 0$$

This corresponds to the following circuit realisation:



Example 1

We will consider the following simple driving point impedance function, where $\omega_1 > 0$ and m = r – 1:

$$Z(s) = \frac{s^2+1}{s(s^2+2)}$$

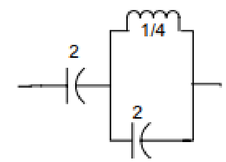
By partial fraction expansion we get:

$$Z(s) = \frac{A}{s} + \frac{Bs}{s^2+2} = \frac{A(s^2+2)+Bs^2}{s(s^2+2)}$$

$$\therefore A+B=1, \quad 2A=1$$

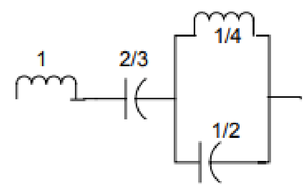
$$A = \tfrac{1}{2}, B = \tfrac{1}{2}$$

This corresponds to the circuit shown:



Example 2 $\quad \omega_1 > 0$ and m = r + 1:

$$Z(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)} = s + \frac{3}{2s} + \frac{s}{2(s^2+2)}$$

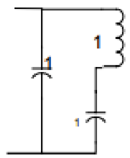


Foster Synthesis

Second Foster Form

Example 1

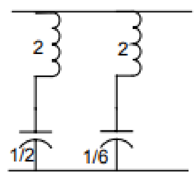
$$Z(s)=\frac{s^2+1}{s(s^2+2)} \Rightarrow Y(s)=\frac{s(s^2+2)}{s^2+1}=s+\frac{s}{s^2+1}$$



Example 1

Example 2

$$Z(s)=\frac{(s^2+1)(s^2+3)}{s(s^2+2)} \Rightarrow Y(s)=\frac{s(s^2+2)}{(s^2+1)(s^2+3)}=\frac{s}{2(s^2+1)}+\frac{s}{2(s^2+3)}$$



Example 2

Thank you for your attention.